Integrating WeBWorK into Textbooks

Sample Exercises

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Abstract

This is a sample article to demonstrate integrating WeBWorK homework problems into content authored with PreTeXt. While technically an example of PreTeXt's article format, it is intended to closely resemble a chapter of a PreTeXt book that is divided into sections.

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Rob Beezer (University of Puget Sound) and Alex Jordan (Portland Community College) worked on the PreTeXt enhancements that make this possible. Mike Gage (University of Rochester), Geoff Goehle (Western Carolina University), and Alex Jordan made this possible by enhancing the WeBWorK end, and generally maintaining WeBWorK software.

This article assumes a mild familiarity with both PreTeXt and WeBWorK. For more information about either project, follow the links.

1 Arithmetic

Some questions with quantitative answers.

Checkpoint 1.1 Adding Single-Digit Integers. A simple, but functional example to begin with. If you are just learning how to add, you can test yourself here.

Compute the sum of 6 and 1:

$6 + 1 = \underline{\hspace{1cm}}$

Answer. 7

Solution. $6 + 1 = 7$.

That was a simple problem. Let’s move on.
If you are familiar with WeBWorK, then it may be a surprise to you to be interacting with a WeBWorK problem this way, without having logged in to WeBWorK.

**Checkpoint 1.2 Declaring a Problem Seed.** You can also declare a seed to specify a version of any problem that has randomization. Here is the same problem (“copied” in the PreTeXt source), but with a seed specified.

Compute the sum of 5 and 8:

$$ 5 + 8 = \underline{\quad} $$

**Answer.** 13

**Solution.** $5 + 8 = 13$.

**Checkpoint 1.3 Controlling Randomness.** You can code your problem with randomization, but then use a specific seed and WeBWorK’s \$envir{problemSeed} to override that randomization for the purposes of the version that will appear in HTML and print output.

Compute the sum of 1 and 2:

$$ 1 + 2 = \underline{\quad} $$

**Answer.** 3

**Solution.** $1 + 2 = 3$.

**Checkpoint 1.4 Special Answer Checking.** One of the strengths of WeBWorK is its ability to give intelligent feedback for incorrect answers.

- There is general feedback for when the student’s answer is in an entirely different ballpark from the correct answer. Try entering something like “y”.

- There is general feedback for when the student’s answer is not in the right form. Try entering “x^2*x^3”, which, right or wrong, is unsimplified.

- And problems can be written to detect and respond to common mistakes. Try entering an answer where you multiply the two exponents (instead of adding them, which would be correct.)

Use the properties of exponents to simplify $x^5 x^3$.

$$ x^5 x^3 = \underline{\quad} $$

**Answer.** $x^8$

**Solution.** We add the exponents as follows, while including a gratuitous reference to the quadratic formula:

$$ x^5 x^3 = x^{5+3} \quad \text{Theorem 2.1} $$

$$ = x^8 $$

**Checkpoint 1.5 Using Hints.** Hints can be inserted into exercises. Whether a hint is visible in the HTML depends on the value of \$showHint in PGcourse.pl in the WeBWorK course that is hosting these exercises. How the hint is displayed in the pdf output is controlled in the usual way that an PTX hint is controlled.

Simplify the expression $\sqrt{48}$.

$$ \sqrt{48} = \underline{\quad} $$

**Hint.** Factor the number inside the radical.

**Answer.** $4 \sqrt{3}$
Solution.

\[ \sqrt{48} = \sqrt{4^2 \cdot 3} = 4\sqrt{3} \]

Checkpoint 1.6 No Randomization. This problem has no randomization at all, not even if it were exported to be part of a problem set on a WeBWorK server. As such, it really doesn’t need any lines of Perl code in its setup, so you have the option of skipping that part of the authoring process.

\[ 1 + 2 = \boxed{} \]

Answer. \( 3 \)

For more about problems that do not require any randomization, see the PTX Author Guide.

2 The Quadratic Formula

In the previous section, we saw relatively simple WeBWorK questions. This section demonstrates how even very complicated WeBWorK problems can still behave well.

Here is a theorem that gives us a formula for the solutions of a second-degree polynomial equation. Note later how the WeBWorK problem references the theorem by its number. This seemingly minor detail demonstrates the degree to which WeBWorK and PreTeXt have been integrated.

Theorem 2.1 Quadratic Formula. Given the second-degree polynomial equation \( ax^2 + bx + c = 0 \), where \( a \neq 0 \), solutions are given by

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Proof.

\[
\begin{align*}
ax^2 + bx + c &= 0 \\
ax^2 + bx &= -c \\
4ax^2 + 4bx &= -4c \\
4ax^2 + 4bx + b^2 &= b^2 - 4ac \\
(2ax + b)^2 &= b^2 - 4ac \\
2ax + b &= \pm \sqrt{b^2 - 4ac} \\
2ax &= -b \pm \sqrt{b^2 - 4ac} \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\end{align*}
\]

Checkpoint 2.2 Solving Quadratic Equations. Consider the quadratic equation \( 4x^2 - 21x - 18 = 0 \).

(a) Identify Coefficients.

Identify the coefficients for the quadratic equation using the standard form from Theorem 2.1.

\[ a = \boxed{4}, \ b = \boxed{-21}, \ c = \boxed{-18} \]

Answer 1. \( 4 \)

Answer 2. \( -21 \)

Answer 3. \( -18 \)
Solution. Take the coefficient of $x^2$ for the value of $a$, the coefficient of $x$ for $b$, and the constant for $c$. In this case, they are $a = 4$, $b = -21$, $c = -18$.

(b) Use the Quadratic Formula.

Using the quadratic formula, solve the equation.

Answer. $\left\{6, -\frac{3}{4}\right\}$

Solution. Recall that the quadratic formula is given in Theorem 2.1. You already identified $a = 4$, $b = -21$, and $c = -18$, so the results are:

$$x = \frac{-(-21) + \sqrt{(-21)^2 - 4 \cdot 4 \cdot (-18)}}{2 \cdot 4} = 6$$

or

$$x = \frac{-(-21) - \sqrt{(-21)^2 - 4 \cdot 4 \cdot (-18)}}{2 \cdot 4} = -\frac{3}{4}$$

This conclusion is just here for testing.

Checkpoint 2.3 Nested tasks. This exercise tests that nested tasks work.

Consider the quadratic equation $5x^2 - 16x - 16 = 0$.

(a) Identify Coefficients.

Identify the coefficients for the quadratic equation using the standard form from Theorem 2.1.

(i) $a = \underline{\hspace{1cm}}$

Answer. 5

Solution. Take the coefficient of $x^2$ for the value of $a$. In this case, $a = 5$.

(ii) $b = \underline{\hspace{1cm}}$

Answer. $-16$

Solution. Take the coefficient of $x$ for the value of $b$. In this case, $b = -16$.

(iii) $c = \underline{\hspace{1cm}}$

Answer. $-16$

Solution. Take the constant term for the value of $c$. In this case, $c = -16$.

(b) Use the Quadratic Formula.

Using the quadratic formula, solve the equation.

Answer. $\left\{4, -\frac{4}{5}\right\}$

Solution. Recall that the quadratic formula is given in Theorem 2.1. You already identified $a = 5$, $b = -16$, and $c = -16$, so the results are:

$$x = \frac{-(-16) + \sqrt{(-16)^2 - 4 \cdot 5 \cdot (-16)}}{2 \cdot 5} = 4$$
This conclusion is just here for testing.

**Checkpoint 2.4 Copy a Problem with Tasks.** We are testing copying the quadratic equation problem above (Checkpoint 2.2), since it is structured with `<task>`, and we also provide a new seed.

Consider the quadratic equation $2x^2 - 5x - 25 = 0$.

(a) **Identify Coefficients.**

Identify the coefficients for the quadratic equation using the standard form from Theorem 2.1.

\[ a = \_\_\_, \ b = \_\_\_, \ c = \_\_\_ \]

**Answer 1.** 2

**Answer 2.** −5

**Answer 3.** −25

**Solution.** Take the coefficient of $x^2$ for the value of $a$, the coefficient of $x$ for $b$, and the constant for $c$. *In this case*, they are $a = 2$, $b = -5$, $c = -25$.

(b) **Use the Quadratic Formula.**

Using the quadratic formula, solve the equation.

**Answer.** \{5, $-\frac{5}{2}$\}

**Solution.** Recall that the quadratic formula is given in Theorem 2.1. You already identified $a = 2$, $b = -5$, and $c = -25$, so the results are:

\[ x = \frac{-(-5) + \sqrt{(-5)^2 - 4 \cdot 2 \cdot (-25)}}{2 \cdot 2} = 5 \]

or

\[ x = \frac{-(-5) - \sqrt{(-5)^2 - 4 \cdot 2 \cdot (-25)}}{2 \cdot 2} = -\frac{5}{2} \]

This conclusion is just here for testing.

We repeat a version of a previous exercise using the deprecated `<stage>` element. This is for testing to monitor if `<stage>` decays.

**Checkpoint 2.5 Solving Quadratic Equations.**

(a) **Identify Coefficients.**

Consider the quadratic equation given by

\[ 6x^2 - 31x - 30 = 0. \]

First, identify the coefficients for the quadratic equation using the standard form from Theorem 2.1.