Integrating WeBWorK into Textbooks

Sample Exercises

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Abstract
This is a sample article to demonstrate integrating WeBWorK homework problems into content authored with PreTeXt. While technically an example of PreTeXt’s article format, it is intended to closely resemble a chapter of a PreTeXt book that is divided into sections.

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Rob Beezer (University of Puget Sound) and Alex Jordan (Portland Community College) worked on the PreTeXt enhancements that make this possible. Mike Gage (University of Rochester), Geoff Goehle (Western Carolina University), and Alex Jordan made this possible by enhancing the WeBWorK end, and generally maintaining WeBWorK software.

This article assumes a mild familiarity with both PreTeXt and WeBWorK. For more information about either project, follow the links.

1 Arithmetic

Some questions with quantitative answers.

Checkpoint 1.1 Adding Single-Digit Integers. A simple, but functional example to begin with. If you are just learning how to add, you can test yourself here.

Compute the sum of 6 and 1:

\[ 6 + 1 = \]

Answer. \( 7 \)

Solution. \( 6 + 1 = 7 \).

That was a simple problem. Let’s move on.
If you are familiar with WeBWorK, then it may be a surprise to you to be interacting with a WeBWorK problem this way, without having logged in to WeBWorK.

**Checkpoint 1.2 Declaring a Problem Seed.** You can also declare a seed to specify a version of any problem that has randomization. Here is the same problem (“copied” in the PreTeXt source), but with a seed specified.

Compute the sum of 5 and 8:

5 + 8 = 

**Answer.** 13

**Solution.** 5 + 8 = 13.

**Checkpoint 1.3 Controlling Randomness.** You can code your problem with randomization, but then use a specific seed and WeBWorK’s $envir\text{(problemSeed)}$ to override that randomization for the purposes of the version that will appear in HTML and print output.

Compute the sum of 1 and 2:

1 + 2 = 

**Answer.** 3

**Solution.** 1 + 2 = 3.

**Checkpoint 1.4 Special Answer Checking.** One of the strengths of WeBWorK is its ability to give intelligent feedback for incorrect answers.

- There is general feedback for when the student’s answer is in an entirely different ballpark from the correct answer. Try entering something like “y”.
- There is general feedback for when the student’s answer is not in the right form. Try entering “x^2*x^3”, which, right or wrong, is unsimplified.
- And problems can be written to detect and respond to common mistakes. Try entering an answer where you multiply the two exponents (instead of adding them, which would be correct.)

Use the properties of exponents to simplify $x^5 \cdot x^3$.

$x^5 \cdot x^3 = 

**Answer.** x^8

**Solution.** We add the exponents as follows, while including a gratuitous reference to the quadratic formula:

$x^5 \cdot x^3 = x^{5+3} \quad \text{Theorem 2.1}$

$= x^8$

**Checkpoint 1.5 Using Hints.** Hints can be inserted into exercises. Whether a hint is visible in the HTML depends on the value of $showHint$ in PGcourse.pl in the WeBWorK course that is hosting these exercises. How the hint is displayed in the pdf output is controlled in the usual way that an PTX hint is controlled.

Simplify the expression $\sqrt{48}$.

$\sqrt{48} = 

**Hint.** Factor the number inside the radical.

**Answer.** $4\sqrt{3}$
Solution.

\[ \sqrt{48} = \sqrt{4^2 \cdot 3} = 4\sqrt{3} \]

**Checkpoint 1.6 No Randomization.** This problem has no randomization at all, not even if it were exported to be part of a problem set on a WeBWorK server. As such, it really doesn’t need any lines of Perl code in its setup, so you have the option of skipping that part of the authoring process.

\[ 1 + 2 = \underline{3} \]

**Answer.** 3

For more about problems that do not require any randomization, see the PTX Author Guide.

# 2 The Quadratic Formula

In the previous section, we saw relatively simple WeBWorK questions. This section demonstrates how even very complicated WeBWorK problems can still behave well.

Here is a theorem that gives us a formula for the solutions of a second-degree polynomial equation. Note later how the WeBWorK problem references the theorem by its number. This seemingly minor detail demonstrates the degree to which WeBWorK and PreTeXt have been integrated.

**Theorem 2.1 Quadratic Formula.** Given the second-degree polynomial equation \( ax^2 + bx + c = 0 \), where \( a \neq 0 \), solutions are given by

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.
\]

**Proof.**

\[
ax^2 + bx + c = 0 \\
ax^2 + bx = -c \\
4ax^2 + 4bx = -4c \\
4ax^2 + 4bx + b^2 = b^2 - 4ac \\
(2ax + b)^2 = b^2 - 4ac \\
2ax + b = \pm \sqrt{b^2 - 4ac} \\
2ax = -b \pm \sqrt{b^2 - 4ac} \\
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

**Checkpoint 2.2 Solving Quadratic Equations.** Consider the quadratic equation \( 4x^2 - 21x - 18 = 0 \).

(a) **Identify Coefficients.**

Identify the coefficients for the quadratic equation using the standard form from Theorem 2.1.

\( a = \underline{\phantom{1}}, \ b = \underline{\phantom{1}}, \ c = \underline{\phantom{1}} \)

**Answer 1.** 4

**Answer 2.** -21

**Answer 3.** -18
Solution. Take the coefficient of $x^2$ for the value of $a$, the coefficient of $x$ for $b$, and the constant for $c$. In this case, they are $a = 4$, $b = -21$, $c = -18$.

(b) Use the Quadratic Formula.

Using the quadratic formula, solve the equation.

Answer. $\{6, -\frac{3}{4}\}$

Solution. Recall that the quadratic formula is given in Theorem 2.1. You already identified $a = 4$, $b = -21$, and $c = -18$, so the results are:

$$x = \frac{-(−21) + \sqrt{(−21)^2 - 4 \cdot 4 \cdot (-18)}}{2 \cdot 4} = 6$$

or

$$x = \frac{-(−21) - \sqrt{(−21)^2 - 4 \cdot 4 \cdot (-18)}}{2 \cdot 4} = -\frac{3}{4}$$

This conclusion is just here for testing.

Checkpoint 2.3 Nested tasks. This exercise tests that nested tasks work.

Consider the quadratic nested tasks. This exercise tests that nested tasks work.

(a) Identify Coefficients.

Identify the coefficients for the quadratic equation using the standard form from Theorem 2.1.

(i) $a = \_\_\_\_\_\_$.

Answer. 5

Solution. Take the coefficient of $x^2$ for the value of $a$. In this case, $a = 5$.

(ii) $b = \_\_\_\_\_\_$.

Answer. $-16$

Solution. Take the coefficient of $x$ for the value of $b$. In this case, $b = -16$.

(iii) $c = \_\_\_\_\_\_$

Answer. $-16$

Solution. Take the constant term for the value of $c$. In this case, $c = -16$.

(b) Use the Quadratic Formula.

Using the quadratic formula, solve the equation.

Answer. $\{4, -\frac{4}{5}\}$

Solution. Recall that the quadratic formula is given in Theorem 2.1. You already identified $a = 5$, $b = -16$, and $c = -16$, so the results are:

$$x = \frac{-(−16) + \sqrt{(−16)^2 - 4 \cdot 5 \cdot (-16)}}{2 \cdot 5} = 4$$
or

\[ x = \frac{-(-16) - \sqrt{(-16)^2 - 4 \cdot 5 \cdot (-16)}}{2 \cdot 5} = -\frac{4}{5} \]

This conclusion is just here for testing.

**Checkpoint 2.4 Copy a Problem with Tasks.** We are testing copying the quadratic equation problem above (Checkpoint 2.2), since it is structured with `<task>`, and we also provide a new seed.

Consider the quadratic equation \(2x^2 - 5x - 25 = 0\).

(a) *Identify Coefficients.*

Identify the coefficients for the quadratic equation using the standard form from Theorem 2.1.

\[ a = \text{ } b = \text{ } c = \text{ } \]

**Answer 1.** 2

**Answer 2.** -5

**Answer 3.** -25

**Solution.** Take the coefficient of \(x^2\) for the value of \(a\), the coefficient of \(x\) for \(b\), and the constant for \(c\). In this case, they are \(a = 2\), \(b = -5\), \(c = -25\).

(b) *Use the Quadratic Formula.*

Using the quadratic formula, solve the equation.

**Answer.** \( \left\{ 5, -\frac{5}{2} \right\} \)

**Solution.** Recall that the quadratic formula is given in Theorem 2.1. You already identified \(a = 2\), \(b = -5\), and \(c = -25\), so the results are:

\[ x = \frac{-(-5) + \sqrt{(-5)^2 - 4 \cdot 2 \cdot (-25)}}{2 \cdot 2} = 5 \]

or

\[ x = \frac{-(-5) - \sqrt{(-5)^2 - 4 \cdot 2 \cdot (-25)}}{2 \cdot 2} = -\frac{5}{2} \]

This conclusion is just here for testing.

We repeat a version of a previous exercise using the deprectated `<stage>` element. This is for testing to monitor if `<stage>` decays.

**Checkpoint 2.5 Solving Quadratic Equations.**

(a) *Identify Coefficients.*

Consider the quadratic equation given by

\[ 6x^2 - 31x - 30 = 0. \]

First, identify the coefficients for the quadratic equation using the standard form from Theorem 2.1.
\[ a = \_, \quad b = \_, \quad c = \_ \]

**Answer 1.** 6  
**Answer 2.** −31  
**Answer 3.** −30

**Solution.** Take the coefficient of \( x^2 \) for the value of \( a \), the coefficient of \( x \) for \( b \), and the constant for \( c \). In this case, they are \( a = 6, \ b = -31, \ c = -30 \).

**(b) Use the Quadratic Formula.**

Using the quadratic formula, solve \( 6x^2 - 31x - 30 = 0 \).

\[ x = \] or \[ x = \]

**Answer 1.** 6  
**Answer 2.** −\( \frac{5}{6} \)

**Solution.** Recall that the quadratic formula is given in Theorem 2.1. You already identified \( a = 6, \ b = -31, \ c = -30 \), so the results are:

\[
x = \frac{-(-31) + \sqrt{(-31)^2 - 4 \cdot 6 \cdot (-30)}}{2(6)} = 6
\]

or

\[
x = \frac{-(-31) - \sqrt{(-31)^2 - 4 \cdot 6 \cdot (-30)}}{2(6)} = -\frac{5}{6}
\]

3 Open Problem Library

The **Open Problem Library** (OPL) is a repository of curated WeBWorK problems. At last count it had something like 30,000 problems, searchable by topic.

Most of the problems in this sample chapter have their source authored within the same document as the narrative. However the problems in this section are from the OPL and elsewhere on the server that is hosting the WeBWorK calculations. For HTML output we can connect to the server and render the problem here in an interactive fashion. For print/PDF we use the `pretext` script to connect to the server to produce PreTeXt code for the problem. We can use the `exercise.text.*` switches to control which parts of the problem (statements, solutions, hints) are incorporated.

Problems do not have to be in the OPL to reside on a server; any problem that is accessible from the WeBWorK course which is hosting the WeBWorK computation may be used this way.

**Checkpoint 3.1 Cylinder Volume.** This problem is one that Portland Community College has donated to the Open Problem Library.

A cylinder’s base’s radius is 6 m, and its height is 10 m.
a. This cylinder’s volume, in terms of \( \pi \), is \( \underline{360\pi} \) m\(^3\).
b. This cylinder’s volume, rounded to the hundredth place, is \( \underline{1130.97} \) m\(^3\).

**Answer 1.** \( 360\pi \) m\(^3\)

**Answer 2.** \( 1130.97 \) m\(^3\)

**Solution.** We use \( r \) to represent the base’s radius, and \( h \) to represent the cylinder’s height.

A cylinder’s volume formula is \( V = \text{(base area)} \cdot \text{height} \). A cylinder’s base is a circle, with its area formula \( A = \pi r^2 \).

Putting together these two formulas, we have a cylinder’s volume formula: 
\[
V = \pi r^2 h
\]

Throughout these computations, all quantities have units attached, and we only show them in the final step.

a. Using the volume formula, we have:
\[
V = \pi r^2 h
= \pi \cdot 6^2 \cdot 10
= \pi \cdot 360
= 360\pi \text{ m}^3
\]
Don’t forget the volume unit m\(^3\).

b. To find the decimal version, we replace \( \pi \) with its decimal value, and we have:
\[
V = 360\pi
\approx 360 \cdot 3.14\ldots
\approx 1130.97 \text{ m}^3
\]
Don’t forget the volume unit m\(^3\).

This uninteresting image, authored in \LaTeX{} syntax, is here only to make sure its automatically-assigned identifier is correct, given that the OPL problem just above is going to generate another image for its static representation.
Warning 3.2 Incompatible Problems. Not all problems that come from the OPL are compatible with PreTeXt. The reasons vary. Perhaps the problem uses older WeBWorK macros that cannot be augmented to provide PreTeXt output. Perhaps the problem nests a table within a table, which PreTeXt will not support. Perhaps there are graphics in the problem that are not sized appropriately for PreTeXt output and there’s nothing you can do about it. And there are many more reasons.

Unfortunately the variety of reasons for which a problem may be incompatible means that at present, there is no way to validate a problem automatically. Using an incompatible problem might not raise any errors at any stage of the PreTeXt processing, but large chunks of the problem may be missing, especially in print output. So we recommend that you review all output modes when using an OPL problem. Chances are that if it is behaving in print, it will behave elsewhere. But that is an unproven theory at this point.

4 Antidifferentiation

4.1 A Few More Features

This subsection demonstrates a few more features.

Definition 4.1 Antiderivative of a Function. Suppose that \( f(x) \) and \( F(x) \) are two functions such that

\[
F'(x) = f(x).
\]

Then we say \( F \) is an antiderivative of \( f \).

The Fundamental Theorem of Calculus in one of the high points of a course in single-variable course.

Theorem 4.2 The Fundamental Theorem of Calculus. If \( f(x) \) is continuous, and the derivative of \( F(x) \) is \( f(x) \), then

\[
\int_a^b f(x) \, dx = F(b) - F(a)
\]

Proof. Left to the reader.

We state an equivalent version of the FTC, which is less-suited for computation, but which perhaps is a more interesting theoretical statement.

Corollary 4.3 Suppose \( f(x) \) is a continuous function. Then

\[
\frac{d}{dx} \int_a^x f(t) \, dt = f(x)
\]  
(4.1)

Proof. We simply take the indicated derivative, applying Theorem 4.2 at (4.2).

\[
\frac{d}{dx} \int_a^x f(t) \, dt = \frac{d}{dx} (F(x) - F(a))
\]  
(4.2)

\[
= \frac{d}{dx} F(x) - \frac{d}{dx} F(a)
\]

\[
= f(x) - 0 = f(x)
\]  
(4.3)

\[\]
4.2 WeBWorK Exercises

The first problem in this list is coming from the WeBWorK Open Problem Library. One implication of this is that we might want to provide some commentary that connects the problem to the text. The other two ask for essay answers, which would be graded by an instructor, so in the HTML output there is no opportunity to provide an answer.

1. **Antiderivatives.** Consult Definition 4.1 and the The Fundamental Theorem of Calculus to assist you with the following problem.

\[ \int_0^5 (5e^x + 3 \sin x) \, dx = \quad \]

**Answer.** 739.214808956493

**Solution.** SOLUTION

\[
\begin{align*}
\int_0^5 (5e^x + 3 \sin x) \, dx &= 5e^x - 3 \cos x \bigg|_0^5 \\
&= (5e^5 - 3 \cos 5) - (5e^0 - 3 \cos 0) \\
&= 5e^5 - 3 \cos 5 - 2
\end{align*}
\]

Unless the OPL has changed this problem out from under us, note the “SOLUTION” appearing in the solution. That is hard-coded into the OPL version of the problem. This is an example of something undesirable that may happen when using OPL problems that were not originally written with PreTeXt in mind.

2. **Every Continuous Function has an Antiderivative.** WeBWorK problems can allow for open-ended essay responses that are intended to be assessed later by the instructor. For anonymous access, no text field is provided. But if this problem were used within WeBWorK as part of a homework set, users could submit an answer.

   Explain how we can use Corollary 4.3 to say that every continuous function *always* has a derivative. (And we will demonstrate here that you can use a macro from docinfo: \( \int_1^2 \frac{1}{x} \, dx = \ln(2) \). It will work in the WeBWorK problem, regardless of whether you are using images, MathJax, or hardcopy.)

3. **Inverse Processes.** “Differentiation and integration are inverse processes.”

   Cite *specific results* from this section in an explanation of how they justify this (somewhat imprecise) claim.

**Exercise Group.** For the given function \( f \), find \( \int f(x) \, dx \).

   Note that these common instructions are phrased in such a way that they would read well if they were applied to only one exercise at a time. That will happen if these exercises are exported as .pg files, for example to be used in online homework from a WeBWorK server.

   4. \( f(x) = \sin(x) \)  
   **Answer.** \( -\cos(x) + C \)

   5. \( f(x) = e^x \)  
   **Answer.** \( e^x + C \)

6. **Show Your Work.** Sometimes you would like a student to give a “simple” answer that WeBWorK can automatically assess, but you would also like the student to show their work or reasoning. Perhaps there is a particular method that you want to see the student use to find the answer. So you have a regular answer blank and also an essay blank. For practical reasons,
you may wish to use the same problem on your WeBWorK server, but omit
the essay part. For example, if you want to use that problem but leave
out the manual grading. For this, WeBWorK has the explanation_box
tool, demonstrated here.

Use the definition of the derivative to find \( \frac{d}{dx} x^2 \).
Show your work.

**Answer.** 2x

## 5 Math Content

This section helps with testing aspects of math content.

**Checkpoint 5.1 Math Elements and Alignment.** In this exercise we
demonstrate the allowed math elements: <m>, <me>, and <md>. The last of
these may have attribute @alignment with options gather, align, or alignat.
The first two are used by default, depending on if you have & or \amp in your
math.

If \( a = 5 \) and \( b = 12 \), then

\[
a^2 + b^2 = 13^2.\]

Here, we solve an equation.

\[
\begin{align*}
2x + 1 &= 3 \\
2x &= 2 \\
x &= 1
\end{align*}
\]

Here we have a three-way inequality to solve.

\[
1 < 2x + 1 < 3 \\
0 < 2x < 2 \\
0 < x < 1
\]

And here, we see a system of equations.

\[
\begin{align*}
2x + 2y + z &= 10 \\
y - 4z &= 9 \\
3z &= -6
\end{align*}
\]

**Checkpoint 5.2 Intertext.** With an <md> you might have <intertext>
among the rows.

Here, we solve an equation.

\[
2x + 1 = 3
\]

Now subtract 1 from each side.

\[
2x = 2
\]

Now divide by 2 on each side.

\[
x = 1
\]

We should also test when the <md> is within a list.

a. Start a list.
b.

\[ 2x + 1 = 3 \]

Now subtract 1 from each side.

\[ 2x = 2 \]

Now divide by 2 on each side.

\[ x = 1 \]

c. Still in the list?

6 PGML Formatting and Verbatim Calisthenics

This section is designed to test various PGML formatting rules and verbatim content returned in answer hashes. Consult the source to see how the special characters and formatting are realized.

Checkpoint 6.1 PGML Formatting. Smart double quotes: “Life is about making an impact, not making an income.”

Smart single quotes: ‘Whatever the mind of man can conceive and believe, it can achieve.’

Regular apostrophes: My siblings’ mother’s daughter isn’t my daughter’s siblings’ mother.

Emphasis: very important
Alert: do not do it
Braces: \{text that looks like a set\}

Some pre-formatted text
with an indented line
and an out-dented line
This should not be altered *a problem*
And this \$NDASH\* should not be an en-dash

Here is some inline code with special characters & < > " ' # $ % ^ _ { } ~ \ * [ ]

\sim \ \star [ ], and here is some

single-line display code with special characters & < > " ' # $ % ^ _ { } ~ \ * [ ]

and here is some

multi-line display
code with special characters & < > " ' # $ % ^ _ { } ~ \ * [ ]

Some raw characters, XML/HTML: & < > " ;
Some raw characters, \TeX: \# $ % \& \_ \{ \} \sim \$
Some raw characters, PGML: \\* \& \{ \} \[ \]
Some characters that need special handling for PGML conversion to HTML or \TeX:

A non breaking space (invisible where a hyphen should be)
An ndash–right here.
An mdash—right here.

Some constructions in “normal” text, which need to be manipulated, lest they get interpreted as PGML markup:

\>>This should not be a centered line<<
Nor a right-justified line, either
We should not get _a phrase in italics_ in the midst of this sentence.

Brackets, unpaired: ] with content between [ 
Brackets, paired, in PGML constructions; \[$NDASH\]*, \[___\]{\$answer}

### This should not be a level 3 heading

--- Not a horizontal rule from three equal signs
== Not a horizontal rule from two equal signs
==== Not a horizontal rule from five equal signs
--- Not a horizontal rule from three hyphens
---- Not a horizontal rule from two hyphens
----- Not a horizontal rule from five hyphens
+ Not an unordered list item
- Not an unordered list item

Here we make MathObject String answers to see how they turn out in answer elements.

**Checkpoint 6.2**

a. Special characters used by XML, character escaping: &’“;
    Now as a MathObject: &’“;

b. Special characters used by LaTeX, where LaTeX \text and MathJax \text disagree: #\&<\text
    Now as a MathObject: #\&<\text

c. Special characters used by LaTeX, where LaTeX \text and MathJax \text can agree: \$
    Now as a MathObject: \$

d. Alphanumeric characters: ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz0123456789
    Now as a MathObject: ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz0123456789

e. Other characters: !()*+.-/:=?@[\]
    Now as a MathObject: !()*+.-/:=?[@]

In answers, because of TeX and MathJax divergence, the first two should come out in verbatim. (And so should any string containing even one of those characters.) The latter three should come out in regular text.

**Answer 1.** &’“;

**Answer 2.** #\&<\text

**Answer 3.** \$

**Answer 4.** ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz0123456789

**Answer 5.** !()*+.-/:=?[@]

Occasionally (and probably very rarely) your PG code will include a string variable where the content is PGML syntax. If this is put into the exercise statement, it will be inserted after PGML processing is done, and therefore it won’t render as you might expect. To insert this content and also have it processed as PGML, use data="pgml". (This is not in the PreTeXt schema as of 3/26/2020, and is subject to change.)

**Checkpoint 6.3** some PGML math: ['\frac{1}{2}+\frac{3}{2}=2']; and some _bold text_ makes:
some PGML math: \( \frac{1}{2} + \frac{3}{2} = 2 \); and some **bold text**

## 7 Multiple Choice

While free-response questions are generally preferred, sometimes the nature of a question lends itself to multiple choice.

**Checkpoint 7.1 Drop-down/Popup.** Note also that the solution to this problem uses an external link.

The number \( \sqrt{2} \) ( □ is □ is not) rational.

**Answer.** is not

**Solution.** If \( \sqrt{2} \) were rational, then \( \sqrt{2} = \frac{p}{q} \), with \( p \) and \( q \) coprime. But then \( 2q^2 = p^2 \). By the Fundamental Theorem of Arithmetic, the power of 2 dividing the left side is odd, while the power of 2 dividing the right side is even. This is a contradiction, so \( \sqrt{2} \) is not rational.

**Checkpoint 7.2 Choose one.** Which of the following suggest that differentiation and integration are inverse processes?

- The Quadratic Formula
- The Fundamental Theorem of Calculus
- The Fundamental Theorem of Arithmetic
- None of these

**Answer.** The Fundamental ... of Calculus

**Solution.** The correct answer is The Fundamental ... of Calculus.

## 8 Tables

Although a WeBWorK problem written in PreTeXt source can’t have a “table” (which would be captioned and cause trouble with free-standing PG output), they can have a “tabular”. Tabulars can be made using the existing syntax and options that PTX offers for table-making, with some exceptions. As of summer 2017, the exceptions are that individual cells cannot have bottom border attributes and individual columns cannot have top border attributes. Also, while PTX table borders are generally minor, medium, or major, when these borders are used in a WeBWorK problem that is rendered as a hard copy problem from within WeBWorK, the only thickness option corresponds to minor. Several more features (like rules in general) do not carry through to static output (print or HTML-preview). However we hope this is only temporary.

**Checkpoint 8.1 Complete this Table.** Complete this multiplication table.

\[
\begin{array}{c|c|c}
\times & 5 & 9 \\
9 & 45 & \\
2 & 18 & \\
\end{array}
\]

**Answer 1.** 45

**Answer 2.** 81

**Answer 3.** 10

**Answer 4.** 18

**Solution.**
9 Graphics in Exercises

It is natural for exercises to have graphics. For example, an exercise might produce a graph of some kind, and ask the reader to extract some information from that graph.

If your WeBWorK server is version 2.16 or later, WeBWorK problems can process TikZ code. Here is an example.

Checkpoint 9.1 A static TikZ graph. This image is a visual proof that $\sum_{k=1}^{n} k$ equals what?

Answer. $C(n + 1, 2)$ or $\frac{(n+1)n}{2}$

Checkpoint 9.2 A randomized TikZ graph. These images may depend on the random seed. In this problem, the height and width of the rectangle are randomized.

Find the area of the rectangle.

Answer. 54 cm$^2$

Checkpoint 9.3 A TikZ graph affected by \texttt{<latex-image-preamble>}. This sample chapter's \texttt{<docinfo>} has a \texttt{<latex-image-preamble>} with \texttt{@syntax} set to \texttt{PGtikz}. This exercise has graph styling that is affected by that.

What are the roots of this polynomial?
Send email user@domain.com.

You gotta give 110%.

I need about $3.50.

Answer. $-3, -2, 3$

Checkpoint 9.4 Special characters. This exercise is to test that special characters behave.

The code below has a printed dollar sign, a printed percent sign, a printed at sign, and a percent sign used as a comment marker.

Send email user@domain.com.

You gotta give 110%.

I need about $3.50.

An older mechanism for creating images is supported and demonstrated here.

Checkpoint 9.5 Solve using a graph. The graph below is a graph of $y = f(x)$. Use the graph to solve the equation $f(x) = 1$.

Answer. $\{1\}$

Solution. The graph reveals that the solution set to $f(x) = 1$ is $\{1\}$.
10 Subject Area Templates

This section samples the subject area template problems found on the WeB-WorK wiki at http://webwork.maa.org/wiki/SubjectAreaTemplates.

Checkpoint 10.1 Answer is a number or a function.

a. Suppose the correct answer is $-2$.

b. Suppose the correct answer is $\frac{8 - 2x^8}{x}$.

Answer 1. $-2$
Answer 2. $\frac{8 - 2x^8}{x}$

Solution. Solution explanation goes here.

Checkpoint 10.2 Answer is a function with domain issues.

a. Suppose the correct answer is $\sqrt{x - 4}$.

b. Suppose the correct answer is $\ln\left(\frac{x}{x - 4}\right)$.

Answer 1. $\sqrt{x - 4}$
Answer 2. $\ln\left(\frac{x}{x - 4}\right)$

Solution. Solution explanation goes here.

Checkpoint 10.3 Multiple choice by popup, radio buttons. My favorite color is □ Red □ Blue □ Green.

My favorite color is

○ Red
○ Blue
○ Green
○ None of these

Answer 1. Blue
Answer 2. Blue

Solution. The correct answer is Blue.

Checkpoint 10.4 There is math in each option for this question. Which expression is not a polynomial?

○ $x$
○ $x^2$
○ $2^x$
Answer. Choice 3

Solution. The answer is Choice 3.

Checkpoint 10.5 Tables. A table with minimal XML source.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>E</td>
<td>F</td>
</tr>
<tr>
<td>G</td>
<td>H</td>
<td>I</td>
</tr>
</tbody>
</table>

Finish this table.

\[
\begin{array}{ccc}
1 & \text{two} & |\pi| \\
I + I & + I + I & \text{six} \\
2^3 - 1 & & \text{VIII}
\end{array}
\]

Answer.  5

Solution. The missing number is 5.

11 Stress Tests

Checkpoint 11.1 PTX problem source with server-generated images.

(a)

(b)
Hint.

Solution.

Checkpoint 11.2 Checking Proper Indentation In Lists. One long exercise, where ordered sublists test the specification of their labels.

a. Simple item
b. Simple item
c. Simple item

And after.
a. Structured item
b. Structured item
c. Structured item

Text before.
a. Structured item
b. Structured item
c. Structured item

And after.
a. Structured item
b.
   A. Sublist Item
   B. Sublist Item
   C. Sublist Item
c. Structured item

Text before.
a. Structured item
b. Text before.
   i. Sublist Item
   ii. Sublist Item
   iii. Sublist Item

And after.
c. Structured item

And after.
a. Structured item
Second paragraph
b.
   I. Sublist Item
      Second paragraph
   II. Sublist Item
      Second paragraph
   III. Sublist Item
      Second paragraph
c. Structured item
Second paragraph

Text before.
a. Structured item
   Second paragraph

b. Text before.
   1. Sublist Item
      Second paragraph
   2. Sublist Item
      Second paragraph
   3. Sublist Item
      Second paragraph

And after.

c. Structured item
   Second paragraph

And after.

a.

\[
1 + 1 = 2
\]

Structured item
   Second paragraph

\[
1 + 1 = 2
\]

b.

a. Sublist

\[
1 + 1 = 2
\]

Item
   Second paragraph

\[
1 + 1 = 2
\]

b.

\[
1 + 1 = 2
\]

Sublist Item
   Second paragraph

\[
1 + 1 = 2
\]

paragrap
c. Sublist Item

\[ 1 + 1 = 2 \]

Second paragraph

c. Structured

\[ 1 + 1 = 2 \]

item

Second paragraph

\[ 1 + 1 = 2 \]

Text before.

a. Structured

\[ 1 + 1 = 2 \]

item

Second paragraph

\[ 1 + 1 = 2 \]

b.

\[ 1 + 1 = 2 \]

Text before.

A. Sublist

\[ 1 + 1 = 2 \]

Item

Second paragraph

\[ 1 + 1 = 2 \]

B.

\[ 1 + 1 = 2 \]

Sublist Item

Second paragraph

\[ 1 + 1 = 2 \]
C. Sublist Item

\[ 1 + 1 = 2 \]
\[ 1 + 1 = 2 \]

Second paragraph
And
\[ 1 + 1 = 2 \]
\[ 1 + 1 = 2 \]
\[ 1 + 1 = 2 \]

Second paragraph
And after.
Text before.
c. Structured item

\[ 1 + 1 = 2 \]
\[ 1 + 1 = 2 \]

Second paragraph
And after.
Text before.
a. Structured

\[ 1 + 1 = 2 \]
\[ 2 = 2 \]

item
Second paragraph

\[ 1 + 1 = 2 \]
\[ 2 = 2 \]

b.

\[ 1 + 1 = 2 \]
\[ 2 = 2 \]

Text before.
i. Sublist

\[ 1 + 1 = 2 \]
\[ 2 = 2 \]

Item
Second paragraph

\[ 1 + 1 = 2 \]
\[ 2 = 2 \]
ii. 

1 + 1 = 2
2 = 2

Sublist Item
Second

1 + 1 = 2
2 = 2

paragraph

iii. Sublist Item

1 + 1 = 2
2 = 2
1 + 1 = 2
2 = 2

Second paragraph
And

1 + 1 = 2
2 = 2

after.
c. Structured item

1 + 1 = 2
2 = 2

1 + 1 = 2
2 = 2

Second paragraph
And after.

Checkpoint 11.3 Checking Proper Indentation In Lists with Images and Tables.

a. Structured item

b.
a. Structured item

b. Structured item
c. Structured item

\[
\begin{array}{c}
a \\
1 \\
d. \\
2^3 - 1
\end{array}
\]

\[
\begin{array}{c}
b \\
1 + I + I + I \\
\pi \}
\end{array}
\]

two 5 six

Structure item

Second paragraph

b. Structured item

**Checkpoint 11.4 Deep-nested lists.** Ordered list.
a. Level 1, first.
b. Level 1, second.
   i. Level 2, first.
   ii. Level 2, second.
      A. Level 3, first.
      B. Level 3, second.
         1. Level 4, first.
         2. Level 4, second.
         3. Level 4, third.
      C. Level 3, third.
   iii. Level 2, third.
c. Level 1, third.

Unordered list.

- Level 1, first.
- Level 1, second.
   - Level 2, first.
   - Level 2, second.
12 Layout Configuration Testing

This section is to provide a single page demonstrating all of the combinations that may factor into a problem’s layout.

12.1 Inline Exercises

Some “inline” exercises, as distinguished from the “divisional” exercises below.

Checkpoint 12.1 $1 + 1 = 2$

Checkpoint 12.2 Has a Title. $1 + 1 = 2$

Checkpoint 12.3 Has an introduction.

Checkpoint 12.4 Has a Title. Has an introduction.

Checkpoint 12.5 $1 + 1 = 2$

Has a conclusion.

Checkpoint 12.6 Has a Title. $1 + 1 = 2$

Has a conclusion.

Checkpoint 12.7 Has an introduction.

Checkpoint 12.8 Has a Title. Has an introduction.

12.2 Divisional Exercises

1. $1 + 1 = 2$

2. Has a Title. $1 + 1 = 2$

3. Has an introduction.

$1 + 1 = 2$
4. **Has a Title.** Has an introduction.
   
   \[1 + 1 = 2\]

5. \[1 + 1 = 2\]
   Has a conclusion.

6. **Has a Title.** \[1 + 1 = 2\]
   Has a conclusion.

7. Has an introduction.
   
   \[1 + 1 = 2\]
   Has a conclusion.

8. **Has a Title.** Has an introduction.
   
   \[1 + 1 = 2\]
   Has a conclusion.

**Exercise Group.** These are inside an exercisegroup.

9. \[1 + 1 = 2\]
10. **Has a Title.** \[1 + 1 = 2\]
11. Has an introduction.
    
    \[1 + 1 = 2\]
12. **Has a Title.** Has an introduction.
    
    \[1 + 1 = 2\]
13. \[1 + 1 = 2\]
   Has a conclusion.
14. **Has a Title.** \[1 + 1 = 2\]
   Has a conclusion.
15. Has an introduction.
    
    \[1 + 1 = 2\]
   Has a conclusion.
16. **Has a Title.** Has an introduction.
    
    \[1 + 1 = 2\]
   Has a conclusion.

---

**A Hints, Answers, and Solutions**

1 · Arithmetic

**Checkpoint 1.1 Adding Single-Digit Integers.**

Answer. \[7\]

Solution. \[6 + 1 = 7\].

**Checkpoint 1.2 Declaring a Problem Seed.**

Answer. \[13\]

Solution. \[5 + 8 = 13\].

**Checkpoint 1.3 Controlling Randomness.**

Answer. \[3\]
Solution. $1 + 2 = 3$.

**Checkpoint 1.4 Special Answer Checking.**

Answer. $x^8$

Solution. We *add* the exponents as follows, while including a gratuitous reference to the quadratic formula:

$$x^5 \cdot x^3 = x^{5+3} \quad \text{Theorem 2.1}$$

$$= x^8$$

**Checkpoint 1.5 Using Hints.**

Hint. Factor the number inside the radical.

Answer. $4\sqrt{3}$

Solution.

$$\sqrt{48} = \sqrt{4^2 \cdot 3} = 4\sqrt{3}$$

**Checkpoint 1.6 No Randomization.**

Answer. $3$

2 · The Quadratic Formula

**Checkpoint 2.2 Solving Quadratic Equations.**

(a) *Identify Coefficients.*

Answer 1. 4

Answer 2. $-21$

Answer 3. $-18$

Solution. Take the coefficient of $x^2$ for the value of $a$, the coefficient of $x$ for $b$, and the constant for $c$. *In this case*, they are $a = 4$, $b = -21$, $c = -18$.

(b) *Use the Quadratic Formula.*

Answer. $\{6, \frac{-3}{4}\}$

Solution. Recall that the quadratic formula is given in Theorem 2.1. You already identified $a = 4$, $b = -21$, and $c = -18$, so the results are:

$$x = \frac{-(-21) + \sqrt{(-21)^2 - 4 \cdot 4 \cdot (-18)}}{2 \cdot 4} = 6$$

or

$$x = \frac{-(-21) - \sqrt{(-21)^2 - 4 \cdot 4 \cdot (-18)}}{2 \cdot 4} = \frac{-3}{4}$$

**Checkpoint 2.3 Nested tasks.**

(a) *Identify Coefficients.*

(i) Answer. $5$

Solution. Take the coefficient of $x^2$ for the value of $a$. *In this case,*

$a = 5$.

(ii) Answer. $-16$
Solution. Take the coefficient of $x$ for the value of $b$. In this case, $b = -16$.

(iii) Answer. $-16$

Solution. Take the constant term for the value of $c$. In this case, $c = -16$.

(b) Use the Quadratic Formula.

Answer. \{4, \frac{-4}{5}\}

Solution. Recall that the quadratic formula is given in Theorem 2.1. You already identified $a = 5$, $b = -16$, and $c = -16$, so the results are:

$$x = \frac{-(-16) + \sqrt{(-16)^2 - 4 \cdot 5 \cdot (-16)}}{2 \cdot 5} = 4$$

or

$$x = \frac{-(-16) - \sqrt{(-16)^2 - 4 \cdot 5 \cdot (-16)}}{2 \cdot 5} = -\frac{4}{5}$$

Checkpoint 2.4 Copy a Problem with Tasks.

(a) Identify Coefficients.

Answer 1. 2
Answer 2. $-5$
Answer 3. $-25$

Solution. Take the coefficient of $x^2$ for the value of $a$, the coefficient of $x$ for $b$, and the constant for $c$. In this case, they are $a = 2$, $b = -5$, $c = -25$.

(b) Use the Quadratic Formula.

Answer. \{5, \frac{-5}{2}\}

Solution. Recall that the quadratic formula is given in Theorem 2.1. You already identified $a = 2$, $b = -5$, and $c = -25$, so the results are:

$$x = \frac{-(-5) + \sqrt{(-5)^2 - 4 \cdot 2 \cdot (-25)}}{2 \cdot 2} = 5$$

or

$$x = \frac{-(-5) - \sqrt{(-5)^2 - 4 \cdot 2 \cdot (-25)}}{2 \cdot 2} = -\frac{5}{2}$$

Checkpoint 2.5 Solving Quadratic Equations.

(a) Identify Coefficients.

Answer 1. 6
Answer 2. $-31$
Answer 3. $-30$
Solution. Take the coefficient of $x^2$ for the value of $a$, the coefficient of $x$ for $b$, and the constant for $c$. In this case, they are $a = 6$, $b = -31$, $c = -30$.

(b) Use the Quadratic Formula.

Answer 1. 6

Answer 2. $-\frac{5}{6}$

Solution. Recall that the quadratic formula is given in Theorem 2.1.

You already identified $a = 6$, $b = -31$, and $c = -30$, so the results are:

\[ x = \frac{-(-31) + \sqrt{(-31)^2 - 4 \cdot 6 \cdot (-30)}}{2(6)} = 6 \]

or

\[ x = \frac{-(-31) - \sqrt{(-31)^2 - 4 \cdot 6 \cdot (-30)}}{2(6)} = -\frac{5}{6} \]

3 · Open Problem Library

Checkpoint 3.1 Cylinder Volume.

Answer 1. $360\pi$ m$^3$

Answer 2. $1130.97$ m$^3$

Solution. We use $r$ to represent the base’s radius, and $h$ to represent the cylinder’s height.

A cylinder’s volume formula is $V = \text{(base area)} \cdot \text{height}$. A cylinder’s base is a circle, with its area formula $A = \pi r^2$.

Putting together these two formulas, we have a cylinder’s volume formula:

$V = \pi r^2 h$

Throughout these computations, all quantities have units attached, and we only show them in the final step.

a. Using the volume formula, we have:

$V = \pi r^2 h$

$= \pi \cdot 6^2 \cdot 10$

$= \pi \cdot 360$

$= 360\pi$ m$^3$

Don’t forget the volume unit m$^3$.

b. To find the decimal version, we replace $\pi$ with its decimal value, and we have:

$V = 360\pi$

$\approx 360 \cdot 3.14\ldots$

$\approx 1130.97$ m$^3$

Don’t forget the volume unit m$^3$.

4 · Antidifferentiation

4.2 · WeBWorK Exercises
4.2.1. Antiderivatives.

Answer.  739.214808956493

Solution. SOLUTION

\[
\int_0^5 (5e^x + 3 \sin x) \, dx = \left. 5e^x - 3 \cos x \right|_0^5 \\
= (5e^5 - 3 \cos 5) - (5e^0 - 3 \cos 0) \\
= 5e^5 - 3 \cos 5 - 2
\]

4.2.4. Answer. \(- \cos(x) + C\)

4.2.5. Answer. \(e^x + C\)

4.2.6. Show Your Work.

Answer. \(2x\)

6 · PGML Formatting and Verbatim Calisthenics

Checkpoint 6.2

Answer 1. &lt;&gt;"";

Answer 2. #%&gt;&lt;\^\_~

Answer 3. \\{

Answer 4. ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz0123456789

Answer 5. !()*+,-./:=?@[]

7 · Multiple Choice

Checkpoint 7.1 Drop-down/Popup.

Answer. is not

Solution. If \(\sqrt{2}\) were rational, then \(\sqrt{2} = \frac{p}{q}\), with \(p\) and \(q\) coprime. But then \(2q^2 = p^2\). By the Fundamental Theorem of Arithmetic, the power of 2 dividing the left side is odd, while the power of 2 dividing the right side is even. This is a contradiction, so \(\sqrt{2}\) is not rational.

Checkpoint 7.2 Choose one.

Answer. The Fundamental ... of Calculus

Solution. The correct answer is The Fundamental ... of Calculus.

8 · Tables

Checkpoint 8.1 Complete this Table.

Answer 1. 45

Answer 2. 81

Answer 3. 10

Answer 4. 18

Solution.

\[
\begin{array}{ccc}
\times & 5 & 9 \\
9 & 45 & 81 \\
2 & 10 & 18
\end{array}
\]

9 · Graphics in Exercises

Checkpoint 9.1 A static TikZ graph.

Answer. \(C(n + 1, 2)\) or \(\frac{(n+1)n}{2}\)
Checkpoint 9.2 A randomized TikZ graph.
Answer. $54 \text{ cm}^2$

Checkpoint 9.3 A TikZ graph affected by \texttt{<latex-image-preamble>}.  
Answer. $-3, -2, 3$

Checkpoint 9.5 Solve using a graph.
Answer. $\{1\}$
Solution. The graph reveals that the solution set to $f(x) = 1$ is $\{1\}$.

![Graph Image]

10 · Subject Area Templates

Checkpoint 10.1 Answer is a number or a function.
Answer 1. $-2$
Answer 2. $\frac{x - 2 - x}{x}$
Solution. Solution explanation goes here.

Checkpoint 10.2 Answer is a function with domain issues.
Answer 1. $\sqrt{x - 4}$
Answer 2. $\ln\left(\frac{x}{x - 1}\right)$
Solution. Solution explanation goes here.

Checkpoint 10.3 Multiple choice by popup, radio buttons.
Answer 1. Blue
Answer 2. Blue
Solution. The correct answer is Blue.

Checkpoint 10.4
Answer. Choice 3
Solution. The answer is Choice 3.

Checkpoint 10.5 Tables.
Answer. 5
Solution. The missing number is 5.

11 · Stress Tests

Checkpoint 11.1 PTX problem source with server-generated images.
(a) Solution.

(b) Hint.

Solution.